Department of Mathematics

Pattamundai College, Pattamundai

6^{th} Semester

Differential Geometry

DSE - 3

Section - A

1. a) What is local differential geometry ?

b) What is Global differential geometry ?

c) Define space core in terms of Intersection of two surfaces ?

d) Define parametric representation of space curve ?

e) State vector representation of space curve ?

f) Define class of a vector valued function.

g) Find the expression for unit tangent vector to a curve.

h) What as the condition that a given curve and a surface have a contact of nth order.

i) What is inflectional tangent ?

j) What is the equation of the tangent line to the curve $x = t$, $y = t^2$, $z = t^3$ at the point $t = 1$

k) Write the equation of tangent line to the curve $\vec{r} = (1+t, -t^2, 1+t^3)$ at $t = 1$

l) Define

i) Principal normal

ii) Binormal

m) What are the orthogonal unit vectors in curves in space.

n) Define normal plane.

o) Define osculating plane.

p) Define rectifying plane.

q) Write the equation of principal normal.

r) Write the equation of binormal.

s) What do you mean by curvature at a point 'P' an a curve ?

P.T.O.

t) Define Torsion ?

u) What is radius of curvature ?

v) What is radius of torsion ?

w) What is screw curvature and express screw curvature in terms of K and $\overline{C}$.

x) What is the necessary and sufficient condition that a given curve is a plane curve ?

y) What is the necessary and sufficient condition that a curve to be a straight line ?

z) What is sphere of curvature ?

2\. a) What is involute ?

b) What is surface ?

c) Give with example parametric equation of a given surface are not unique ?

d) What is Mange's form of the surface ?

e) Define class of a surface ?

f) Find the equation of the tangent place are normal to the surface $xyz = 4$ at the point (1, 2, 2)

g) Find a unit normal vector to the surface $2xz^2 - 3xy - 4x = 7$ at the point (1, –1, 2)

h) Define Envelope ?

i) What as edge of regression ?

j) Define first fundamental form of the surface ? Why it is called metric.

k) What is the relation between E, F, G ?

l) The metric is invariant under a transformation of parameters (T/F)

m) What is the angle between the parametric curves ?

n) What is second fundamental form of the surface ?

o) Write down the relations between the fundamental magnitudes of second order ?

p) What is the necessary condition that a surface is a developable surface ?

q) Define

i) Osculating developable

ii) Polar developable

iii) Rectifying developable

r) Define Helix ?

Section - B

1. a) Find the equation of the osculating plane of the curve given by

$\vec{r} = (a \sin t + b \cos t, a \cos t + b \sin t, c \sin 2t)$

b) Evaluate curvature of the cubic curve given by $\vec{r} = (u, u^2, u^3)$

c) Find the torsion of the helix

$x = a \cos\theta, y = a \sin \theta, z = a\theta \tan \alpha$

d) For the curve $x = 4a \cos^3 u, y = 4a \sin^3 u,$

$z = 3c \cos 2u$ prove that $k = \dfrac{a}{6(a^2 + c^2) \sin 2u}$

e) Prove that the necessary condition for a curve to be a helix is that its curvature and torsion are in a constant ratio.

f) Derive curvature at a point 'P' as, $K = |\vec{r}''|$

g) Derive terms at a point 'P' as, $\tau = |\hat{b}|$

h) Show that the necessary condition for the curve to be a plane curve is $\begin{bmatrix} \vec{r}^{I} & \vec{r}^{II} & \vec{r}^{III} \end{bmatrix} = 0$

i) Find the radius of curvature of the helix $x = a \cos\theta$, $y= a \sin \theta$, $z = a \theta \tan \alpha$

j) Find the radius of torsion of the helix $x = a \cos \theta$, $y = a \sin \theta$, $z = a \theta \tan \alpha$

k) Prove that the locus of the centre of curvature is an evolute only when the curve as plane.

l) Find the equation of tangent surface to the curve $\vec{r} = (u, u^2, u^3)$

m) Show that the sum of the squares of the intercepts on the coordinate axes made by the tangent plane to the surface $x^{2/3} + y^{2/3} + z^{2/3} = a^{2/3}$ is constant

n) Prove that the tangle-plane to the surface $xyz = a^3$ and the Coordinate planes bound a tetrahedron of constant volume.

o) Find the envelope of the plane $3xt^2 - 3yt + z = t^3$?

p) Find the envelope of the $lx + my + nz = p$ when $a^2 \rightarrow a^2l^2+b^2m^2+c^2n^2-p^2=0$

q) Find the envelope of the normal planes drawn through the generators of the cone

$ax^2 + by^2 + cz^2 = 0$

r) Show that the metric is invariant under a transformation of parameters.

s) Show that the metric $\sqrt{2}\, du^2+2Fdudu+Gdu^2$ is a positive definite quadratic form in du, dv.

t) Find the 1st order fundamental magnitudes for the surface of revolution

$x = u \cos v,\ y = u \sin v,\ z = f(u)$

u) Find the 2nd order fundamental magnitudes of the curve $x = a(u+v),\ y = b(u-v),\ z = uv$

v) Calculate the fundamental magnitudes for the Mange's form of the surface $z = f(x,y)$

w) Calculate the fundamental magnitudes and the unit normal for the helicoid

$x = u \cos v,\ y = u \sin v,\ z = f(u) + cv$

x) Prove that a real surface for which the equations $\frac{E}{L} = \frac{F}{M} = \frac{G}{N}$ hold is either plane or spherical.

y) Find the angle between two directions (tagent direction) on the surface at a point P having direction coefficients (l,m) and (l', m')

z) Find the angle between directions with diruction coefficients (l,m) and the curve v = constant.

2. a) Show that through every point of the surface there passes one and only one member of the curves $\phi(u,v) = c$

b) Show that every family of curves on a surface possesses orthogonal trajectories.

c) Show that the curves bisecting the angle between the parametric curves are given by $Edu^2 - Gdv^2 = 0$

d) Show that the curves $du^2-(u^2+a^2)\,dv^2 = 0$ form an orthoganal system on the right helicoid

$\vec{r} = (u \cos v, u \sin v, av)$

e) Define the following with examples

i) Umblic

ii) First curvature

iii) Mean curvature

iv) Gaussian curvature

f) Define the differential equation of lines of curvature through a point on the surface $z = f(x,y)$

g) Prove that the necessary and sufficient condition that a curve on a surface be a line of curvature is that

$K d\vec{r} + d\vec{N} = 0$ at each of its point

h) Prove that the necessary condition that the parametric curves be lines of curvature are F = o and M = 0

i) Prove that the sum of normal curvatures to any point on a surface in two directions at right angles is constant and equal to sum of the principal curvatures at that point.

j) Show that the parametric curves are asymptotic lines on the surfaces

$x = u \cos v,\ y = u \sin v,\ z = cv.$

k) Show that on the surface

$x = a\,(u+v),\ y = b\,(u-v),\ z = uv$ the parametric curves are asymptotic

l) Find the asymptotic lines an the Catenard of revolution $u = c \cosh z/c$.

m) Find the axymptotic lines of the conoid $x = u \cos v,\ y = u \sin v,\ z = f(u)$

n) Prove that the assumption lives of the catenoid $u = c \cos h\, z/c$ lie and the cylinders $zu = c\,(al^{\theta} + a^{-1}e^{-\theta})$ where a is arbitrary.

Section - C

1. a) State and prove serrate frenal formula

b) For the helix x = a cos t, y = a sin t, z = at tan α, prove that $\frac{ds}{dt}$ = a sec α and that the length of the curve measured from the point t = 0 is at sec α

c) For the curve $x = 3t$, $y = 3t^2$, $z = 2t^3$ show that any plane meets it in three points and deduce the equation of the osculating plane at $t^2\ t^1$

d) Find the osculating plane at any plane of the equation of curve gain by the equating $x = 4a \cos^2 t$, $y^2\ 4a \sin^2 t$, $z = 3c \cos 2t$.

e) Show that the curve x = t, y = $\frac{1+t}{t}$, $t^2\ \frac{1-t^2}{t}$ lies an a plane.

f) Find $f(\theta)$ so that the curve $x = a \cos\theta$, $y = a \sin\theta$, $z = f(\theta)$ determines a plane curve.

g) For the curve

$x = a(3a - u^3)$, $y = 3au^2$, $z = a(3a + u^3)$ show that the curvature and torsion are equal.

h) For the curve $x = 3u$, $y = 3u^2$, $z = 2u^3$ prove that $\ell = -6 = \frac{3}{2}(1+2u^2)^2$

i) For the curve

$x = 4a \cos^3 u$, $y = 4a \sin^3 u$, $z = 3c \cos 2u$

Prove that $k = \frac{a}{6(a^2+c^2)\sin 2a}$

j) Find the osculating plane, curvatures and torsion at any point of the curve

$x = a \cos 2u$, $y = a \sin 2u$, $z = 2a \sin u$.

k) Find the radii of curvature and torsion at any point of the curve $x^2 + y^2 = a^2$, $x^2 - y^2 = az$?

l) Prove that the curvature and torsion are both constant then the curve is circular helix.

m) Prove that x = a (u+v), y = b (u–v), z = uv the parametric curves are straight lines. Also calculate the fundamental magnitudes.

n) Show that the caves $du^2 - (u^2+a^2)\,du^2 = 0$ form an orthoganal system the right helicoid

$\vec{r} = (u \cos v,\ u \sin v,\ av)$

o) Show that the necessary and suffricut condition for a curve on the surface to be a line of curvature is that the normal to the surface at any two consecutive points of the curve intersect

p) State and prove Euler's theorem.

q) Show that on the surface $z = f(x,y)$ the asymptosic lines are $r dx^2+2s\, dxdy+t dy^2 = 0$ and their torsions are $\pm\sqrt{s^2 - rt} / (1 + p^2 + q^2)$

r) prove that as points common to the surface $a(yz+zx+xy) = xyz$ and a sphere whose centre is origin, the tangent plane to the surface wakes intercepts on the axes whose sum is constant.

s) Prove that the curve of intersection of two surfaces is a line of curvature on both the surfaces then the surfaces cut at a constant angle. conversely if two surfaces cut at a constant angle and the curve of inter section is a line of convature on one of them then it is also a line of curvature on the other.

t) Show that the differential equation of the lines of curvature can be put in the form $\left[\vec{N}, d\vec{N}, d\vec{r}\right] = 0$

u) Prove that the necessary and sufficient condition for a surface to be developable surface is that its Gaussian convature is zero.

v) Prove that the necessary and sufficient condition for the surface given by $z = f(x,y)$ to represent a developable surface is $rt - s^2 = 0$, Also verify by using the surface.

i) $xy = (z - c)^2$

ii) $xyz = a^3$

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